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Zagreb Indices of Triple Square Snake Graphs

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Abstract

Several lattice structures which can also be thought as graphs have been shown to be useful in the study of large networks. In this work, we study 15 topological graph indices from the class of Zagreb indices of three interesting lattice structures called as square snake graphs, double square snake graphs and triple square snake graphs. We use vertex and edge partitions of these graphs and calculate their indices by means of these partitions.

1 Introduction

[1] [2]

Let G be a graph. We denote its vertex set by $V(G)$ and edge set by $E(G)$. Let $n = |V(G)|$ be the order of G and let $m = |V(G)|$ be the size of G. If $v \in V(G)$, then the degree of v in G is denoted by $d_G(v)$ and is defined to be the number of vertices in G which are adjacent to v. If there is no risk of confusion, we briefly use dv instead of $d_G(v)$.

Snake graphs are studied in different fields of mathematics and other branches. They have finite or infinite one or two dimensional repetitions of some geometric shape. They are planar bipartite graphs. In [2, 3, 4, 5], the authors studied snake graphs in relation with cluster algebras. In [8], Shiffler constructed snake graphs consisting of square tiles and studied them in relation with perfect matchings and positive continued fractions which are used in estimating real numbers by some infinite sequences of rational numbers. Therefore the idea of continued fractions have been a popular and useful area of number theory. They are used in the solutions of some Diophantine equations. Bradshaw et al. continued the above work in [1] and established their relations with

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linear algebra by studying their characteristic polynomials in relation with the Chebycheff polynomials of the first and second type. In [6], snake graphs are studied in relation with strongly*-graphs.

Recall that a square snake graph is a graph denoted by $C^1_{4,k}$ as seen in Fig. 1.

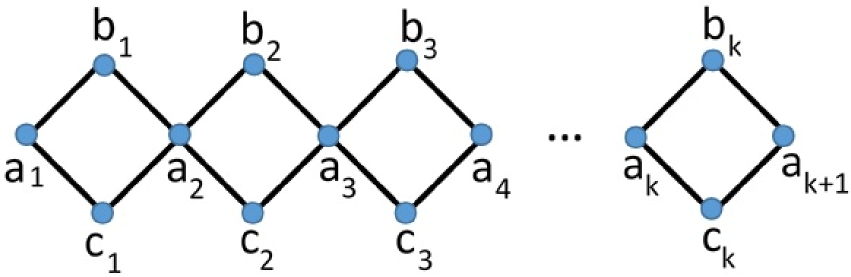

Figure 1 The square snake graph $C^1_{4,k}$

In this work, we consider triple square snake graphs $T(C^1_{4,k})$ as a generalization of square snake graphs as in Fig. 2.

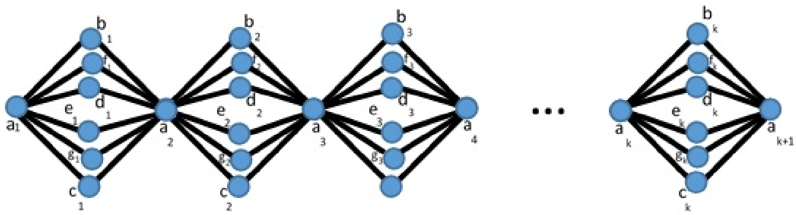

Figure 2 The triple square snake graph $T(C^1_{4,k})$

A topological index is a mathematical formula calculated by means of either vertex degrees, distances or some graph parameters. Topological graph indices of square snake graphs were studied in [7]. In this work, we will determine some Zagreb type topological graph indices of the triple square snake graphs $T(C^1_{4,k})$ by means of vertex degrees. The following indices are used in this work:

The first and second Zagreb indices are

$$M_1(G) = \sum_{v \in V(G)} dv^2 \text{ and } M_2(G) = \sum_{uv \in E(G)} dudv.$$

The forgotten index is

$$F(G) = \sum_{v \in V(G)} dv^3.$$

The multiplicative first and second Zagreb indices are defined by

$$\prod_1(G) = \prod_{v \in V(G)} dv^2$$

and

$$\prod_2(G) = \prod_{u,v \in E(G)} du \cdot dv.$$

The generalized first and second Zagreb indices are

$$M_1^{\alpha}(G) = \sum_{v \in V(G)} dv^{\alpha}$$

and

$$M_2^{\alpha}(G) = \sum_{uv \in E(G)} (dudv)^{\alpha}.$$

The generalized sum connectivity index is defined by

$$H_{\alpha}(G) = \sum_{uv \in E(G)} (du + dv)^{\alpha}.$$

The redefined first, second and third Zagreb indices are

$$ReZG_1(G) = \sum_{uv \in E(G)} \frac{du+dv}{dudv},$$

$$ReZG_2(G) = \sum_{uv \in E(G)} \frac{dudv}{du+dv}$$

and

$$ReZG_3(G) = \sum_{uv \in E(G)} dudv(du + dv).$$

The second Gourava index is defined by

$$M_{r,s}(G) = \sum_{u,v \in E(G)} [du^r \cdot dv^s + du^s \cdot dv^r].$$

The reformulated first, second Zagreb indices and reformulated forgotten index are defined by

$$RM_1(G) = \sum_{u,v \in E(G)} [d(uv)^2],$$

$$RM_2(G) = \sum_{e,e' \in E(G)} [de \cdot de']$$

and

$$RF(G) = \sum_{u,v \in E(G)} [d(uv)^3].$$

2 Main results

Now we are ready to determine the above listed topological graph indices in the class of Zagreb type indices of the triple square snake graph $T(C^1_{4,k})$. Our main result is as follows:

Theorem 2.1. *Some Zagreb type topological graph indices of the triple square snake graph* $T(C^1_{4,k})$ *are as follows:*

$$\begin{aligned}
M_1(T(C^1_{4,k})) &= 24(7k-3),\\
M_2(T(C^1_{4,k})) &= 144(2k-1),\\
F(T(C^1_{4,k})) &= 48\cdot(37k-27),\\
\prod\nolimits_1(T(C^1_{4,k})) &= 2^{16k}\cdot 3^{2k+2},\\
\prod\nolimits_2(T(C^1_{4,k})) &= 2^{12(3k-1)}\cdot 3^{12k},\\
M_1^{\alpha}(T(C^1_{4,k})) &= 2^{\alpha+1}(3k+3^{\alpha})+(k-1)\cdot 2^{2\alpha}\cdot 3^{\alpha},\\
M_2^{\alpha}(T(C^1_{4,k})) &= 12^{\alpha}[(k-1)\cdot 2^{\alpha}+1],\\
H_{\alpha}(T(C^1_{4,k})) &= 12\cdot 8^{\alpha}+12(k-1)\cdot 14^{\alpha},\\
ReZG_1(T(C^1_{4,k})) &= 7k+1,\\
ReZG_2(T(C^1_{4,k})) &= \tfrac{18}{7}(8k-1),\\
ReZG_3(T(C^1_{4,k})) &= 576(7k-5),\\
M_{r,s}(T(C^1_{4,k})) &= 12\cdot 2^{r+s}[3^r+3^s+(k-1)(6^r+6^s)],\\
RM_1(T(C^1_{4,k})) &= 432(4k-3),\\
RM_2(T(C^1_{4,k})) &= 216(44k-35),\\
RF(T(C^1_{4,k})) &= 2592(8k-7).
\end{aligned}$$

Proof. The triple square snake graph $T(C^1_{4,k})$ has $7n+1$ vertices and $12n$ edges. The vertex degrees are 2, 6 or 12 and the vertex partition of $T(C^1_{4,k})$ is given in Table 1 where d_i in the first column denotes the degree $d(v_i)$ of the vertex v_i and the numbers in the second column gives the number of vertices v_i having vertex degree d_i:

Table 1: Vertex partition of $T(C^1_{4,k})$

d_i	$\sharp\, v_i$
2	$6k$
6	2
12	$k-1$

Also the edge partition of $T(C^1_{4,k})$ is shown in Table 2:

Let us start with calculating the first Zagreb index. By the vertex partition of the triple square

Table 2: Edge partition of $T(C^1_{4,k})$

(d_i, d_j)	$\sharp\,(v_i, v_j)$
(2,6)	12
(2,12)	$12(k-1)$

snake graph $T(C^1_{4,k})$ given in Table 1, we obtain

$$\begin{aligned} M_1(T(C^1_{4,k})) &= \textstyle\sum_{v\in V(G)} dv^2 \\ &= 6k\cdot 2^2 + 2\cdot 6^2 + (k-1)\cdot 12^2 \\ &= 24(7k-3). \end{aligned}$$

We next calculate the second Zagreb index of the triple square snake graph $T(C^1_{4,k})$. From the edge partition of this graph given in Table 2, we have

$$\begin{aligned} M_2(T(C^1_{4,k})) &= \textstyle\sum_{uv\in E(G)} du\cdot dv \\ &= 12(2\cdot 6) + 12\cdot(k-1)\cdot(2\cdot 12) \\ &= 144(2k-1). \end{aligned}$$

The forgotten Zagreb index of the triple square snake graph $T(C^1_{4,k})$ is calculated as below:

$$\begin{aligned} F(T(C^1_{4,k})) &= \textstyle\sum_{v\in V(G)} dv^3 \\ &= \textstyle\sum_{u,v\in E(G)}[du^2 + dv^2] \\ &= 6k\cdot 2^3 + 2\cdot 6^3 + (k-1)\cdot 12^3 \\ &= 48(37k-27). \end{aligned}$$

Alternatively, using the edge partition table, we have

$$\begin{aligned} F(T(C^1_{4,k})) &= \textstyle\sum_{u,v\in E(G)}[du^2 + dv^2] \\ &= 12(2^2+6^2) + 12(k-1)(2^2+12^2) \\ &= 48(37k-27). \end{aligned}$$

Let us continue with the first multiplicative Zagreb index. By Table 1, we get

$$\begin{aligned} \textstyle\prod_1(T(C^1_{4,k})) &= \textstyle\prod_{v\in V(G)} dv^2 \\ &= 2^{12k}\cdot 6^4\cdot 12^{2k-2} \\ &= 2^{16k}\cdot 3^{2k+2}. \end{aligned}$$

Similarly, the second multiplicative Zagreb index of the triple square snake graph is obtained as

$$\begin{aligned} \textstyle\prod_2(T(C^1_{4,k})) &= \textstyle\prod_{u,v\in E(G)} du\cdot dv \\ &= \textstyle\prod_{v\in V(G)} du^{dv} \\ &= 2^{12k}\cdot 6^{12}\cdot 12^{12(k-1)} \\ &= 2^{12(3k-1)}\cdot 3^{12k}. \end{aligned}$$

The general first Zagreb index of the triple square snake graph $T(C^1_{4,k})$ is similarly obtained as follows:

$$\begin{aligned} M^\alpha_1(T(C^1_{4,k})) &= \sum_{v\in V(G)} dv^\alpha \\ &= 6k\cdot 2^\alpha + 2\cdot 6^\alpha + (k-1)\cdot 12^\alpha \\ &= 2^{\alpha+1}(3k+3^\alpha) + (k-1)\cdot 2^{2\alpha}\cdot 3^\alpha. \end{aligned}$$

Also using the edge partition table, we alternatively have

$$\begin{aligned} M^\alpha_1(T(C^1_{4,k})) &= \sum_{u,v\in E(G)} [du^{\alpha-1} + dv^{\alpha-1}] \\ &= 12\cdot[2^{\alpha-1} + 6^{\alpha-1}] + 12(k-1)[2^{\alpha-1} + 12^{\alpha-1}] \\ &= 2^{\alpha+1}(3k+3^\alpha) + (k-1)2^{2\alpha}\cdot 3^\alpha. \end{aligned}$$

The general second Zagreb index of the triple square snake graph $T(C^1_{4,k})$ is

$$\begin{aligned} M^\alpha_2(T(C^1_{4,k})) &= \sum_{u,v\in E(G)} [du\cdot dv]^\alpha \\ &= 12(2\cdot 6)^\alpha + 12(k-1)(2\cdot 12)^\alpha \\ &= 12^{\alpha+1} + (k-1)\cdot 12^{\alpha+1}\cdot 2^\alpha \\ &= 12^{\alpha+1}[(k-1)2^\alpha + 1]. \end{aligned}$$

The general sum connectivity index of the graph $T(C^1_{4,k})$ is

$$\begin{aligned} H_\alpha(T(C^1_{4,k})) &= \sum_{u,v\in E(G)} [du+dv]^\alpha \\ &= \sum_{u,v\in E(G)} [d(uv)+2]^\alpha \\ &= 12(2+6)^\alpha + 12(k-1)(2+12)^\alpha \\ &= 12\cdot 8^\alpha + 12(k-1)\cdot 14^\alpha. \end{aligned}$$

The redefined first Zagreb index of the triple square snake graph $T(C^1_{4,k})$ is

$$\begin{aligned} ReZG_1(T(C^1_{4,k})) &= \sum_{u,v\in E(G)} \\ &= \sum_{u,v\in E(G)} [\tfrac{du+dv}{du\cdot dv}] \\ &= 7k+1. \end{aligned}$$

The redefined second Zagreb index of the triple square snake graph $T(C^1_{4,k})$ is

$$\begin{aligned} ReZG_2(T(C^1_{4,k})) &= \sum_{u,v\in E(G)} [\tfrac{du\cdot dv}{du+dv}] \\ &= 12\tfrac{12}{8} + 12(k-1)\tfrac{24}{14} \\ &= \tfrac{18}{7}\cdot(8k-1). \end{aligned}$$

The redefined third Zagreb index of the triple square snake graph $T(C^1_{4,k})$ is

$$\begin{aligned} ReZG_3(T(C^1_{4,k})) &= \sum_{u,v\in E(G)} [du\cdot dv]\cdot[du+dv] \\ &= (12\cdot 8)\cdot 12 + (14\cdot 24)\cdot 12(k-1) \\ &= 576(7k-5). \end{aligned}$$

The second Gourava index of the triple square snake graph $T(C^1_{4,k})$ is

$$\begin{aligned} M_{r,s}(T(C^1_{4,k})) &= \sum_{u,v\in E(G)}[du^r \cdot dv^s + du^s \cdot dv^r],\ \forall\, r,s \in R \\ &= 12[2^r \cdot 6^s + 2^s \cdot 6^r] + 12(k-1)[2^r \cdot 12^s + 2^s \cdot 12^r] \\ &= 12 \cdot 2^{r+s}[3^r + 3^s + (k-1)(6^r + 6^s)]. \end{aligned}$$

The reformulated first Zagreb index of the triple square snake graph $T(C^1_{4,k})$ is

$$\begin{aligned} RM_1(T(C^1_{4,k})) &= \sum_{u,v\in E(G)}[d(uv)^2] \\ &= \sum_{u,v\in E(G)}[du + dv - 2]^2 \\ &= 12[2+6-2]^2 + 12(k-1)[2+12-2]^2 \\ &= 432(4k-3). \end{aligned}$$

The reformulated second Zagreb index of the triple square snake graph $T(C^1_{4,k})$ is

$$\begin{aligned} RM_2(T(C^1_{4,k})) &= \sum_{e,e'\in E(G)}[de \cdot de'] \\ &= (6\cdot 6)\cdot 30 + (6 \cdot 12)\cdot 12 + (12\cdot 12)\cdot 66(k-1) \\ &= 216(44k-35). \end{aligned}$$

The reformulated third (forgotten) Zagreb index of the triple square snake graph $T(C^1_{4,k})$ is

$$\begin{aligned} RF(T(C^1_{4,k})) &= \sum_{u,v\in E(G)} d(uv)^3 \\ &= (2+6-2)^3 \cdot 12 + (2+12-2)^3 \cdot 12(k-1) \\ &= 2592(8k-7). \end{aligned}$$

□

3 Summary and conclusions

In this work, triple square snake graphs are considered as special network structures and their Zagreb type topological graph indices are calculated. The same calculations can easily be carried out to multiple square snake graphs with many chemical applications.

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